

PA STAMP
(State Tax Analysis Modeling Program)

A Complete Tax Model for Pennsylvania State

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Suffolk University

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TABLE OF CONTENTS

1. INTRODUCTION TO CGE MODELING	3
Tax Experiment.....	3
2. WHAT IS PA STAMP?.....	4
Constructing a CGE model	6
3. THE PA STAMP: THE MODEL OUTLINED	8
Organizing the Data	8
Industrial sectors	8
Factor Sectors.....	9
Household Sectors	9
Investment Sector.....	9
Government Sectors.....	10
Rest of the World.....	12
4. PA STAMP: THE MODEL IN DETAIL	13
DETAILED EQUATIONS FOR PA STAMP.....	13
HOUSEHOLD DEMAND.....	13
LABOR SUPPLY	17
MIGRATION.....	17
THE BEHAVIOR OF PRODUCERS/FIRMS	18
TRADE WITH OTHER STATES AND COUNTRIES.....	21
INVESTMENT	23
GOVERNMENT.....	25
ELASTICITY ASSUMPTIONS FOR PA STAMP	33

TABLE OF TABLES

FIGURE 1. CIRCULAR FLOW DIAGRAM	5
TABLE 1. GOVERNMENT SECTORS	10
TABLE 2. INDUSTRY ELASTICITIES	34
TABLE 3. HOUSEHOLD-RELATED ELASTICITIES	36

1. INTRODUCTION TO CGE MODELING

Changes in tax rates have measurable effects on taxable activities. The weight of evidence shows that state-level tax increases have significant negative effects on state economic activity.¹ Yet it is not easy to quantify these effects, and the job can only be done satisfactorily with the help of a complete tax model.

In order to be able to analyze sweeping changes in the Pennsylvania tax system, the Beacon Hill Institute (BHI) has built a *Computable General Equilibrium* (CGE) model of Pennsylvania, called “PA STAMP”. STAMP stands for “State Tax Analysis Modeling Program”. In this report, we explain the concept behind the CGE model and set out the individual components used in the model. The model’s purpose is to answer questions about what would happen to the Pennsylvania economy under a variety of tax changes chosen by the user.

CGE models are typically large, complex, and difficult to build; for instance, PA STAMP has 3,800 variables and almost a thousand lines of computer code, and every run of the model produces 920 pages of output. A CGE tax model for California was a forerunner of the Model developed by BHI.² California developed its CGE model with state funding, after passing a law (SB 1837, 1994) requiring the Department of Finance to perform “dynamic revenue analysis” of any proposed legislation with a revenue impact of \$10 million or more. In this context, a dynamic revenue analysis differs from a static revenue analysis in that it takes account of the secondary effects of tax changes; for instance, a lower property tax might leave more money in people’s pockets and so, as they spend more, revenue from the sales tax might rise, offsetting in part the initial cut in the property tax.

Tax Experiment

While an econometric model could be used to estimate effects of incremental changes in the levels of existing taxes, PA STAMP is the more appropriate tool when analyzing the effect of major changes at a high level of detail, or when evaluating new taxes (for which the lack of

¹ *Who Benefits from State and Local Economic Development Policies?* Timothy Bartik, Upjohn Institute, 1991.

² P. Berck, E. Golan and B. Smith, with J. Barnhart and A. Dabalén. *Dynamic Revenue Analysis for California*. Summer 1996. University of California at Berkeley and California Department of Finance. On the Web at <http://www.dof.ca.gov:8080/html/fs%5Fdata/dyna%2Drev/dynrev.htm> .

historical evidence makes econometric estimation impossible). In the sections that follow we first provide a brief description of CGE models, and then set out the way in which we built the model for Pennsylvania. The key equations of the model are presented in detail in Section 4.

2. WHAT IS PA STAMP?

PA STAMP is a comprehensive model of the Pennsylvania economy, designed to capture the principal effects of state tax changes on that economy. PA STAMP is a five-year dynamic computable general equilibrium (CGE) tax model. As such, it provides a mathematical description of the economic relationships among producers, households, government and the rest of the world. It is general in the sense that it takes all the important markets and flows into account. It is an equilibrium model because it assumes that demand equals supply in every market (goods and services, labor and capital); this is achieved by allowing prices to adjust within the model (i.e., prices are endogenous). The model is computable because it can be used to generate numeric solutions to concrete policy and tax changes, with the help of a computer. And it is a tax model because it pays particular attention to identifying the role played by different taxes.³

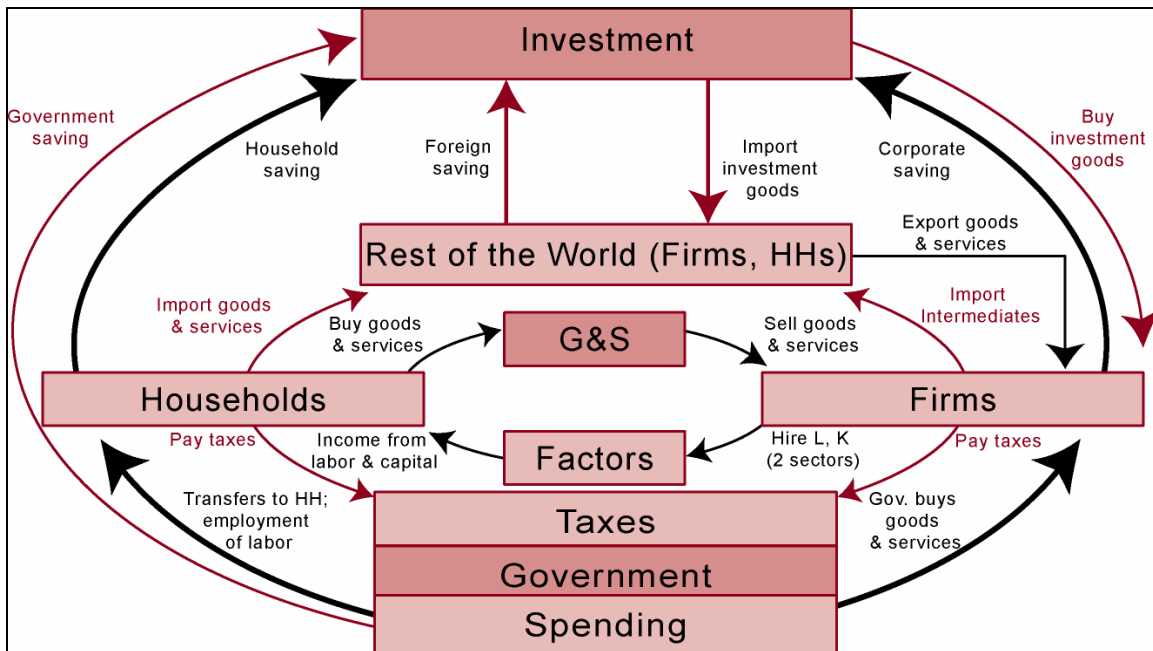
We begin by distinguishing between producers and consumers. Consumers/households earn income by supplying labor (wages and salaries) and capital (dividends and interest); they also receive transfer payments such as pensions. They are assumed to maximize their utility, which they do by using income to buy goods and services, pay taxes and save. Their spending decisions are strongly influenced by the structure of prices they face, and the amount of labor that they are willing to provide depends, to a substantial degree, on the wage rates that they face.

Producers/firms buy inputs (labor, capital and intermediate goods that are produced by other firms) and transform them into outputs. Producers are assumed to maximize profits and are likely to change their decisions about how much to buy or produce depending on the prices they face for inputs and outputs.

³ For a clear introduction to CGE tax models, see John B. Shoven and John Whalley, "Applied General-Equilibrium Models of Taxation and International Trade: An Introduction and Survey," *Journal of Economic Literature*, XXII (September, 1984), 1008. Shoven and Whalley have also written a useful book on the practice of CGE modeling entitled *Applying General Equilibrium* (Cambridge: Cambridge University Press, 1992).

In addition, there is a government sector that collects taxes and fees and provides services and transfers. The rest-of-the world sector consists of the entire world outside of Pennsylvania. The relationships between these components are set out in the circular flow diagram shown in Figure 1.⁴ The arrows in the diagram represent flows of money (for instance, households purchase goods and services), and flows of goods and services (for instance, households supply their labor to firms). The separate box for government shows the flows of funds to government in the form of taxes, as well as government purchases of goods and services and government hiring of labor and capital.

Figure 1. Circular Flow Diagram



Complex as it may seem, the diagram in Figure 1 is still too simple, because it lumps all households into one group, and all firms into another. To provide further detail it is necessary to create *sectors*; PA STAMP has 81 economic sectors. Each sector is an aggregate that groups together segments of the economy. We separate households into seven income classes and firms into 27 industrial sectors. In addition, we distinguish between 30 types of taxes and funds (four at the federal level, 14 at the state level, and 12 at the city level) and 13 categories of government spending (two at the federal level, six at the state level, and five at the city level). To complete the model, there are two factor sectors (labor, capital), an investment sector and a sector that

⁴ Based on a similar diagram in Berck et al., *Dynamic Revenue Analysis for California*.

represents the rest of the world. The choice of sectors was dictated by the availability of suitably disaggregated data (for households and firms), and the purposes of the model.

Sub-national models, such as PA STAMP, are similar in many ways to national and international CGE models. However, they differ in a number of important respects, which are as follows:

- a. In a national model, most saving goes toward domestic investment; however, this need not be true at the regional level. If citizens of Pennsylvania save more than they spend, then the excess saving will leak out of the state.
- b. The smaller the unit under consideration, the greater the importance of trade with the rest of the world. This is an important consideration for state models.
- c. Migration is likely to be larger and more responsive across cities and states than across nations.
- d. In sub-national models, taxes are interdependent. So, for instance, the amount of revenue collected by the Federal personal income tax depends significantly on whether there is a state or local income tax (which may be deducted from income before computing the Federal tax).
- e. Data are less available at the sub-national than national level. This explains why scores of national CGE models have been built, but relatively few sub-national models.

Constructing a CGE model

The construction of a CGE model involves several steps. First, one needs to organize the data needed by the model. PA STAMP starts with data for a single fiscal year, 2004, which we use as a basis to develop a steady state path through fiscal year 2010 in the model. This steady state path is attained by applying growth rates for investment, population, employment and inflation throughout the time frame of the model. In PA STAMP, the investment growth rate is assumed to be 1.31%.⁵ The growth rate for population is assumed to be 1.31%.⁶ The inflation growth rate is assumed to be 3.00%.⁷ To attain a reasonable steady state path, the data for the base year, fiscal year 2006, must be very detailed. Most of the data are organized into a *Social Accounting Matrix (SAM)*, which in this case consists of an 81 by 81 matrix that accounts for the main economic and fiscal flows in the state.

⁵ This figure is derived from taking the average nominal US gross domestic investment for the period 1929-2002 as published by the Bureau of Economic Analysis.

⁶ This figure is the Census projection for PA for the period 2005-2010.

⁷ This figure is based on data obtained from the U.S. Bureau of Labor Statistics.

The model also requires some additional information – for instance, data on employment and on the structure of the Federal income tax – which are put in separate files. And the model requires information on “elasticities;” these are the parameters, typically taken from the academic literature, that measure the responsiveness of households to changes in prices and wages, and of firms to changes in input costs and output prices. These are set out in detail in Section 4 of this report. The economy is assumed to be competitive, and to run at full employment (by which we mean that there is no involuntary unemployment).

Second, the model needs to be specified in detail; the next section of this report sets out details of the model that we constructed for Pennsylvania, along with some comments explaining the choices made at each step.

The third step is to program the model. For this we used the specialized GAMS (General Algebraic Modeling System) software. In order to make the model easier to use, we also developed an interface in Microsoft Excel. This allows the user to enter tax changes on an Excel spreadsheet, click the “Estimate CGE” button, and read the key output on the same spreadsheet; the heavy-duty computing occurs in the background.

Before use, the model must be calibrated. Calibration consists of running the model – i.e., asking it to solve for all the variables in such a way as to maximize (and minimize!) total personal income.⁸ The results for the base year are checked to see that they correspond with the actual values of the variables in the SAM. Once the model reproduces the base year values, it is considered calibrated. Calibration is an important step, as it is essentially a way of checking that the model is working properly.

After it has been calibrated, the model is ready to be used to quantify tax change effects. The procedure is straightforward: specify a new tax rate (or change in the tax), run the model, and compare the new results with the steady state ones. At this point it is also possible to test the sensitivity of the results to different assumptions – such as the values of elasticities – that are incorporated into the model. It is worth stressing that PA STAMP is a policy model and not a

⁸ The choice of variable to maximize has no substantive importance, and is a device for getting the model to solve.

forecasting model; in other words it is designed to answer “what if?” questions, not to estimate what is actually expected to occur in coming years

3. THE PA STAMP: THE MODEL OUTLINED

Organizing the Data

The starting point in building a CGE model is to determine the degree of detail that is desired and to organize the collected data into the useful format of a Social Accounting Matrix (SAM) for the base year. The SAM that we developed for Pennsylvania State is an 81 by 81 matrix. Each of the 5,929 cells in the matrix represents the dollar value of a flow from one sector of the economy to another – for instance, purchases of business services by the utilities sector, or labor earnings flowing to middle-income households. Reading along a row, one finds the payments received by that sector; reading down a column, one sees the payments made by that sector. The SAM is balanced, which means that the sum of the entries in any given row equals the sum of the entries in the corresponding column. Thus, for instance, the revenue received by utilities must equal spending by that sector, so that all incoming and outgoing funds are completely accounted for.

For PA STAMP, we distinguish 27 industrial sectors, two factors (labor and capital), seven household categories, an investment sector, 43 government sectors (26 for taxes, 13 for spending, four government funds) and a sector for the rest of the world. In sectoring the economy we sought to strike a balance between providing a high level of detail (especially on the tax side) and keeping the model to a manageable size. An additional limitation is that the lack of finely disaggregated data limits the degree of detail that is possible. Data availability also determined some of the choices we made; for instance, it is possible to get a breakdown of households into seven income categories (see below for further details), and while we might have preferred a different set of categories, we were constrained by the nature of the data available.

Industrial sectors

Although data for 49 sectors were actually available from the Bureau of Economic Analysis, PA STAMP contains only 27 industrial sectors. This is because some sectors were too small to merit separate attention. In these cases, we combined some industries, such as textiles and apparel. In other cases, there were no matching employment figures, and so it was easier to work with aggregates.

Factor Sectors

We distinguish between two factors, labor and capital (which includes land). Businesses pay wages and salaries to labor, and they generate profits. These are then distributed to household owners as factor income.

Household Sectors

In PA STAMP, households receive wages, capital income and transfers and they use this income to buy goods and services to pay taxes; and to save. In the core of the model, we distinguish seven household sectors, which group households by their levels of income. Expenditure data are available for households in each of these categories, which make it relatively straightforward to work with this structure. One purpose of this disaggregation of households is to allow one to trace the distributive effect of tax changes and another one is to allow different groups to have different levels of sensitivity to labor market conditions.

Even though the core of the model holds only seven household sectors and because the tax experiments may affect a particular sector within one income group – e.g. a personal income tax increase may be directed to people with incomes between \$80,000 and \$90,000 and those people fall in the core model’s \$75,000 to \$100,000 income bracket – PA STAMP provides an interface with a larger number of income brackets. In order to aggregate those brackets back into the core brackets, PA STAMP uses not only the distribution of the tax paid by the disaggregated brackets, but also the distribution of filers in those disaggregated brackets.

Investment Sector

There is one investment/savings sector. Households save, both directly out of their cash incomes, and indirectly because they own shares in businesses that save and reinvest profits. The government also saves and invests. Information is available from the Bureau of Economic Analysis (BEA) on the pattern of gross investment by destination (i.e., how much gross investment went into adding to the stock of capital in utilities, in industry, and so on). We have constructed measures of the capital stock in each sector, and by applying published depreciation rates and adding gross investment, arrived at the capital stock in the subsequent period. This permits the model to track the expansion of the economy over time. The BEA has also produced a matrix, built for the U.S. for 1997, which maps investment by destination with investment by

source. This mapping allows one to determine, for example, how much of the investment destined for utilities is spent on purchasing goods and services from the construction sector and the transport sector. Thus if investment rises, it is possible to identify which sectors would face an expansion in the demand for their output.

Government Sectors

PA STAMP was designed primarily to analyze the effects of major changes in the structure of State taxes, and so we have paid particular attention to providing sufficient detail for government transactions. The sectoring is summarized below in Table 1.

Table 1. Government Sectors

Federal Government Receipts		
USSSTX	Social Security (OASDI and MEDICARE)	Receives payments from employers and households; pays out transfers to households.
USPITX	Federal Personal Income Tax	Receives payments from households, which are put into the Federal normal spending account.
USCITX	Federal Corporation Income Tax	Receives payments from corporations and channels them into the Federal normal spending account.
USOTTX	Other Federal Taxes	Includes excises on motor fuel, alcohol, and tobacco; estate and gift taxes. Also funneled into the Federal normal spending account.
Federal Government Expenditure		
USNOND	Federal Normal Spending	Federal government purchases goods and services, hires labor, and transfers money to PA and to Federal defense fund.
USDEFF	Federal Defense Spending	Purchases goods and services, and pays labor for military purposes.
PA State Government Receipts		
STPITX	PA State Personal Income Tax	Revenues go into PA State general fund.
STSATX	PA State Sales Tax	Revenues go into PA State general fund.
STCITX	PA State Corporate Net Income Tax	Revenues go into PA State general fund.
STGRTX	PA State Gross Receipts Tax	Revenues go into PA State general fund..
STFUTX	PA State Tax on Motor Fuel	Revenues go into PA State motor license fund.
STALTX	PA State Alcohol Beverage Taxes	Revenues go into PA State general fund.
STTCTX	PA State Tax on Cigarettes and Tobacco	Revenues go into PA State general fund.
STKFTX	PA State Capital Stock and Foreign Franchise Tax	Revenues go into PA State general fund.
STOTTX	PA State Other Taxes	Revenues go into PA State general fund.
STMOTX	PA State Motor Vehicle Tax	Revenues go into PA State motor license fund.
STIHTX	PA State Inheritance Tax	Revenues go into PA State general fund.

STINTX	PA State Insurance Premiums Tax	Revenues go into PA State general fund.
STWKTX	PA State Workers Compensation Tax	Sector combines workers compensation and unemployment funds. Receipts go into proprietary fund.
STFEES	PA State Fees, License Permits and Other Revenue	Revenues go into all funds.
STGENF	PA State General Fund	An accounting device. Tax revenue is channeled into this fund before being distributed to other uses.
STFSCF	PA State Federal Special Fund	An accounting device. Tax revenue is channeled into this fund before being distributed to other uses.
STSPCF	PA State Special Funds	An accounting device. Tax revenue is channeled into this fund before being distributed to other uses.
PA State Government Expenditure		
STGGSP	PA State General Spending	General government spending.
STEDUC	PA State Spending on Education	Mainly purchases of goods and services and labor in the higher education sector.
STHELT	PA State Spending on Health & Welfare	Buys some services; mainly transfers funds to local health spending fund.
STPBSF	PA Public Safety	Public safety and fire departments spending.
STTRAN	PA State Spending on Transport	Mainly buys engineering services and construction.
STOTHS	PA State Other Spending	Miscellaneous other spending by the state on labor, goods and services.
Local Government Receipts		
LOPRTX	Local Tax on Residential Property	Revenues go into the local education, health, public safety, transportation and other funds.
LOPBTX	Local Tax on Business Property	Revenues go into the local education, health, public safety, transportation and other funds.
LOOTTX	Local Other Taxes	Revenues go into the local education, health, public safety, transportation and other funds.
Local Government Expenditure		
LOEDUC	Local Spending on Education	Purchases goods and services and (mainly) pays teacher salaries.
LOHELT	Local Spending on Health	Purchases goods and services and pays labor; large transfers to the poorest category of households.
LOPBSF	Local Public Safety	Public safety and fire departments local spending.
LOTRAN	Local Spending on Local Transportation	Mainly buys engineering services and construction.
LOOTHS	Local Other Spending	Includes spending on police and firefighters, road repair, and miscellaneous local government services.

The Pennsylvania government collects revenue from taxes and fees. Specific tax categories at the city level included in the model are: sales and use, cigarettes and tobacco, mortgage recording,

corporate and personal incomes, and taxes both on residential and commercial properties. The rest of the state taxes are grouped into a residual category (other local taxes).

The revenues from the taxes go to either the PA general fund, the PA capital projects fund or to other funds, or a combination of them. Funds then allocate the money into the five spending categories: education, health and welfare, transportation, public safety or others. A main beneficiary of PA's education expenditure is the Pennsylvania State University system.

Rest of the World

To complete the model, we have included a sector for the rest of the world (ROWSCT). This refers to the world outside of Pennsylvania, i.e., the rest of the United States and other countries. Information on flows between Pennsylvania and the rest of the world is difficult to piece together, and is an area where considerable professional judgment was required.

4. PA STAMP: THE MODEL IN DETAIL

This section of the report explains the PA STAMP model in detail. First, we introduce each equation, providing some context and a short description. Then we present each equation in mathematical form, provide information on the sources of data used, and summarize the elasticity assumptions used in the model.

DETAILED EQUATIONS FOR PA STAMP

PA STAMP is a dynamic CGE model which assumes a steady state growth path. Absent from any “shocks”, the economy is assumed to remain on this path. If the economy experiences a shock, such as a tax change, the economy will diverge from this steady state path and eventually turn onto a new path. The size and length of the divergence will depend on the size of the shock to the economy. Below we set out the equations used in PA STAMP and the assumptions inherent in them.

HOUSEHOLD DEMAND

Households are assumed to maximize their well being (“utility”) by picking baskets of goods and services, subject to their budget constraints. The key set of equations in this section is labeled *Private Consumption*, and consists of a set of demand functions. These demand functions, based on a Cobb-Douglas utility function, take on the simple form,

$$X_{t,i} = \lambda_i * \frac{I_t}{P_{t,i}}, \quad i = 1, \dots, n; t = 1, \dots, n,$$

where $X_{t,i}$ is the quantity demanded of good i at time t , $P_{t,i}$ is the price of good i at time t , I_t is income at time t , and λ_i are parameters that measure the share of income that is devoted to good i . This is the simplest specification that is theoretically satisfactory: it is additive (so spending equals income less taxes less saving), has downward-sloping demand (ensuring that when the price of a good rises, the quantity demanded falls), is zero degree homogeneous in prices and income (so that if prices and incomes were to double, the quantity demanded would not change), and meets the technical requirement of symmetry of the Slutsky matrix. More complex formulations are possible, but there is a lack of reliable data on the elasticity parameters that would be needed in such cases.

Household Gross Factor Income

Comments: The gross income of households in each of the seven groups (indexed by h in the set H) is found by first summing factor income (y_f) from labor and capital, subtracting the social security contributions paid by employees, and then allocating the total to each group on the basis of fixed shares. Factor payments are allocated to each household group using the same fixed shares as were found in the base year. We used historical data to estimate the approximate percentage of income earned in Pennsylvania is earned by non-residents (i.e., commuters from New Jersey, Delaware, Ohio, etc.), and took account of this in the model.

$$\text{Eq. 1.} \quad y_{t,h} = \sum_{f \in F} \frac{\alpha_{h,f} a_{t,h}^w}{\sum_{h \in H} \alpha_{h,f} a_{t,h}^w} y_{t,f} \left(1 - FFP_f\right) \left(1 - \sum_{g \in GF} \tau_{t,g,f}^{jh}\right) \quad \forall t \in T, h \in H, f \in F$$

Description: Household income is the sum of income from each factor (labor and capital) less factor taxes, distributed by household groups according to their share of total.

Data: The information on earnings for each household group comes from PA IMPLAN (an economic impact modeling system which allows users to perform in-depth regional analysis. See <http://www.implan.com> for more details).

Household Disposable Income

Comments: Disposable household income is gross income, less taxes on household income and property (mainly personal income tax (USPITX, STPITX) and residential property tax (LOPRTX)), plus transfer payments (such as social security and unemployment benefits).

$$\text{Eq. 2.} \quad y_{t,h}^d = y_{t,h} - \sum_{g \in GI} t_{t,g,h} a_{t,h}^{hh} - \sum_{g \in GH} \tau_{t,g,h}^h a_{t,h}^{hh} + \sum_{g \in G} w_{hg} a_{t,h}^n \tau_{t,h,g}^{pc} \quad \forall h \in H, t \in T$$

Description: Disposable household income is the household income less income taxes and other household taxes (property taxes etc), plus the government transfer payments.

Private Consumption Expenditure

Comments: This is the simplest demand system that is consistent with theoretical first principles, and it requires only a limited number of parameters.

$$\text{Eq. 3. } c_{t,i,h} = \bar{c}_{t,i,h} \left(\frac{y_{t,h}^d}{\bar{y}_{t,h}^d} \div \frac{p_{t,h}}{\bar{p}_{t,h}} \right)^{\beta_{ih}} \prod_{i' \in I} \left[\frac{p_{t,i'}}{p_{t,i'}} \frac{\left(1 + \sum_{g \in GS} \tau_{t,g,i'}^c \right)}{\left(1 + \sum_{g \in GS} \tau_{t,g,i'}^q \right)} \right]^{\lambda_{i'i}} \quad \forall i \in I, h \in H, t \in T$$

Description: Consumption is a function of baseline consumption, adjusted to reflect the change in household disposable income (in constant prices), and the change in after-tax prices.

Data: By construction, this equation has zero cross price elasticities. In the absence of adequate estimates of demand elasticities we follow the approach taken by Berck et al., setting all income and own-price elasticities equal to unity.

Direct household purchases of imports

Comments: Some household spending goes directly to buy goods and services outside Pennsylvania.

$$\text{Eq. 4. } m_{t,h} = \bar{m}_{t,h} \left(\frac{y_{t,h}^d}{\bar{y}_{t,h}^d} \div \frac{p_{t,h}}{\bar{p}_{t,h}} \right)^{\eta_h^m} \quad \forall h \in H, t \in T$$

Description: Household imports will increase with the increase in disposable income, in constant prices.

Household Savings

Comments: In PA STAMP, household savings is the residual after spending and taxes have been subtracted from income. Thus savings are seen as occurring passively.

$$\text{Eq. 5.} \quad s_{t,h} = y_{t,h}^d - \sum_{i \in I} c_{t,i,h} p_{t,i} \left(1 + \sum_{g \in GS} \tau_{t,g,i}^c \right) - m_{t,h} \quad \forall h \in H, t \in T$$

Description: See comments above.

Data: The savings rates for households at each income level were adjusted based on professional judgement, to account for the imputed savings by corporations (which indirectly represents savings by the owners of the corporations).

Consumer Price Index

Comments: The price index in the reference period is set equal to 1. There is a separate price index for each household group. This allows one to compute the real (rather than nominal) income for each household group. For instance, a tax on foodstuffs would tend to hit poor households relatively hard, and the CPI for poor households would pick up this effect.

$$\text{Eq. 6.} \quad p_{t,h} = \frac{\sum_{i \in I} p_{t,i} \left(1 + \sum_{g \in GS} \tau_{t,g,i}^c \right) c_{t,i,h}}{\sum_{i \in I} \bar{p}_{t,i} \left(1 + \sum_{g \in GS} \tau_{t,g,i}^q \right) c_{t,i,h}} \quad \forall h \in H, t \in T$$

Description: Price index by household group is a function of the baseline price index, adjusted by the change in after-tax prices by industry, according to their corresponding share of consumption.

Data: The consumption of each good by each household group (c_{ih}) is derived from PA IMPLAN. The model also generates some of its own values.

LABOR SUPPLY

Comments: In PA STAMP we model the labor participation rate, defined as the proportion of households in any given income category that work. The participation rate is assumed to rise if wage rates rise, if the taxes levied on earnings fall, or if the transfer payments paid out per non-working household fall. The participation rate for low-income households is assumed to be highly sensitive to the level of transfer payments, but relatively insensitive to changes in taxes or the wage rate. On the other hand, high-income households are assumed to respond substantially to changes in the taxes and wage rates they face.

$$\text{Eq. 7. } a_{t,h}^w = \bar{a}_{t,h}^w \frac{a_{t,h}^{hh}}{\bar{a}_{t,h}^{hh}} \left(\frac{r_{t,L}^a (1 - \tau_{t,h})}{\bar{r}_{t,L}^a (1 - \bar{\tau}_{t,h})} \div \frac{p_{t,h}}{\bar{p}_{t,h}} \right)^{\eta_h^s} \left(\frac{\sum_{g \in G} \frac{w_{t,h,g}}{p_{t,h}}}{\sum_{g \in G} \frac{\bar{w}_{t,h,g}}{\bar{p}_{t,h}}} \right)^{\eta_h^p} \quad \forall t \in T, h \in H$$

Description: The supply of labor is a function of the baseline supply of labor adjusted by population growth, the net change in after income tax wages, and government transfer payments. We used professional judgment in determining the proper elasticities for each household group.

Data: The data on working households by income class came from PA IMPLAN.

MIGRATION

Population

Comments: The number of households in each income group depends first and foremost on the initial number of households. To this we add the natural growth of the population and net in-migration. Migration in turn depends on the level of after-tax income, and the proportion of households that are not working (which reflects the employment prospects facing new migrants). This formulation is in the spirit

of the migration model popularized by Harris and Todaro (*American Economic Review*, 1973).

Eq. 8.

$$a_{t,h}^{hh} = \bar{a}_{t,h}^{hh} + \bar{a}_{t,h}^i \left(\frac{y_{t,h}^d}{a_{t,h}^{hh}} \div \frac{\bar{y}_{t,h}^d}{\bar{a}_{t,h}^{hh}} \div \frac{p_{t,h}}{\bar{p}_{t,h}} \right)^{\eta_h^{yd}} \left(\frac{a_{t,h}^n}{a_{t,h}^{hh}} \div \frac{\bar{a}_{t,h}^n}{\bar{a}_{t,h}^{hh}} \right)^{\eta_h^u} - \bar{a}_h^o \left(\frac{\bar{y}_{t,h}^d}{\bar{a}_{t,h}^{hh}} \div \frac{y_{t,h}^d}{a_{t,h}^{hh}} \div \frac{\bar{p}_{t,h}}{p_{t,h}} \right)^{\eta_h^{yd}} \left(\frac{\bar{a}_{t,h}^n}{\bar{a}_{t,h}^{hh}} \div \frac{a_{t,h}^n}{a_{t,h}^{hh}} \right)^{\eta_h^u}, \forall h \in H, t \in T$$

Description: See comments above.

Data: The elasticities used in this equation are the same as those used for California by Berck et al. (1996), and “reflect the middle ground found in the literature about migration” (p.117).

Number of Non-Working Households

Comments: This is a simple accounting equation; the number of non-working households is the total number of households, less the number that are working.

Eq. 9.

$$a_{t,h}^n = a_{t,h}^{hh} - a_{t,h}^w \quad \forall h \in H, t \in T$$

Description: See comments above.

THE BEHAVIOR OF PRODUCERS/FIRMS

Producers are assumed to maximize profit. Combining intermediate inputs with labor and capital produces output. The amount of intermediate inputs required per unit of output is fixed, but firms have considerable leeway to vary the amounts of capital and labor that they use in production. The value of output less intermediate inputs is value added, and it is useful to compute a price for this value added; it is this price that determines factor demand – i.e. drives firms to hire more or less labor and capital. The amounts of labor and capital inputs, in turn, drive the total value of output via the production function.

Intermediate Demand

Comments: Intermediate goods constitute a fixed share of the value of production.

$$\text{Eq. 10.} \quad v_{t,i} = \sum_{i' \in I} \alpha_{t,i,i'} q_{t,i'} \quad \forall i \in I, t \in T$$

Description: See comments above.

Data: From the PA input-output table, derived from data from IMPLAN, which in turn are based on data from by the Bureau of Economic Analysis.

Production Function

Comments: Output is determined by the quantities of labor and capital used in production; it is assumed that enough intermediate goods will be available. We use a Constant Elasticity of Substitution (CES) production function, which allows a degree of substitution between labor and capital; in other words, if the price of labor rises, firms will cut back on the number of workers they hire, and use more capital instead.

$$\text{Eq. 11.} \quad q_{t,i} = \gamma_{t,i} \left[\sum_{f \in F} \alpha_{t,f,i} (u_{t,f,i}^d)^{-\rho_i} + g \alpha_{t,i} (gk_t)^{-\rho_i} \right]^{-1/\rho_i} \quad \forall i \in I, t \in T$$

Description: In addition to labor and capital used in production, we account for infrastructure.

Data: We use values for the elasticity of substitution that are close to, but slightly lower than, one. This is relatively standard in CGE models. Information on the shares of labor and capital in production come from the Bureau of Economic Analysis.

Price of Value Added

Comments: Define value-added as the value of output less the cost of intermediate inputs. One may then define a “price” of value added, which we then use below in the factor demand (i.e. labor demand, capital demand) equations.

$$\text{Eq. 12.} \quad p_{t,i}^{va} = p_{t,i}^d - \sum_{i' \in I} \alpha_{t,i',i} p_{t,i'} \left(1 + \sum_{g \in GS} \tau_{t,g,i'}^v \right) \quad \forall i \in I, t \in T$$

Description: Price of value-added by industry is the domestic price by industry minus the production prices by industry according to their share in domestic supply, including taxes on intermediates.

Data: Prices are set equal to unit in the baseline case.

Factor Demand

Comments: It is possible to construct a profit function that expresses profits as a function of factor inputs. Microeconomic theory shows that the partial first derivative of the profit function, with respect to a given factor demand variable, gives the demand equation for that factor. The left hand side of the equation shows payments to labor (including the cost of factor taxes such as the employer share of social security contributions). The right hand side gives the amount of value added attributable to the factor. There are separate equations for labor and for capital, for each of the 27 industrial sectors.

$$\text{Eq. 13.} \quad r_{t,f,i} r_{t,f}^a \left(1 + \sum_{g \in GF} \tau_{t,f,g,i}^x \right) u_{t,f,i}^d = p_{t,i}^{va} q_{t,i} \alpha_{t,f,i} \quad \forall i \in I, f \in F, t \in T$$

Description: The factor demand at the current intra-industry rental rate (for labor and capital) times the overall rental rate, including factor taxes is a function of the price of value-added times the industry domestic supply.

Data: Information on the wage bills comes from the Bureau of Economic Analysis. The total wage bill is divided by the number of workers (from the Bureau of Labor Statistics) to get measures of wage rates by industry. The intersectoral wage differentials are not allowed to vary within the model. The cost of capital was derived as property-type income divided by the capital stock. The capital stock was constructed by disaggregating the national aggregate level of capital using a series of proxy measures; further details of the methodology are provided

in Appendix 2 of the *Texas State Tax Analysis Modeling Program: Texas-STAMP* (1999) and although this refers to Texas, the same approach was taken in computing the capital stock for Pennsylvania.

Factor Income

Comments: The total income accruing to factors – i.e. to labor and capital – is computed here.

$$\text{Eq. 14.} \quad y_{t,f} = \sum_{i \in I} r_{t,f,i} r_{t,f}^a u_{t,f,i}^d + \sum_{g \in G} r_{t,f,g} r_{t,f}^a u_{t,f,g}^d \quad \forall f \in F, t \in T$$

Description: The factor income is the sum of factor demand times rental rates, for all industries and government sectors.

TRADE WITH OTHER STATES AND COUNTRIES

From a Pennsylvania perspective, the “rest of the world” consists of the remainder of the United States as well as the world outside the U.S. Goods produced in Pennsylvania are assumed to be close, but not perfect, substitutes for goods produced elsewhere. Thus if prices rise in Pennsylvania, the state’s exports will fall and its imports will rise, but the adjustment need not be very large. There is no need for trade to be balanced; capital flows simply adjust to cover the gap between exports and imports. In this section we also develop a measure of the average price faced by domestic households and firms for goods and services produced by each industry, the price is a weighted average of the price of locally produced and imported goods.

Demand for Exports

Comments: Exports depend on the price of goods within Pennsylvania relative to the price outside the state. If the domestic price rises relative to the foreign price, exports will fall. Note that the elasticity here is negative.

$$\text{Eq. 15.} \quad e_{t,i} = \bar{e}_{t,i} \left[\frac{p_{t,i}^d \div \bar{p}_{t,i}^w}{1 + \sum_{g \in G} \tau_{t,g,i}^m} \right]^{\eta_i^e} \quad \forall i \in I, t \in T$$

Description: Current exports are a function of baseline exports adjusted by the change in domestic prices versus fixed world prices.

Data: The trade data for Pennsylvania are not particularly reliable; we have used our judgement, combined with BEA data, to arrive at sensible estimates. The elasticities we use are similar to those employed by Berck et al.

Domestic Share of Domestic Consumption

Comments: The demand for imports is handled indirectly, by modeling the share of domestic consumption that is supplied by domestic firms (d), following the approach pioneered by Armington (1969). This share depends on the domestic price relative to the price of the same goods in the rest of the world. We ignore import tariffs on the grounds that they are a tiny fraction (less than 1%) of the value of goods imported into Pennsylvania.

$$\text{Eq. 16.} \quad d_{t,i} = \bar{d}_{t,i} \left[\frac{p_{t,i}^d \div \bar{p}_{t,i}^w}{1 + \sum_{g \in G} \tau_{t,g,i}^m} \right]^{n_i^d} \quad \forall i \in I, t \in T$$

Description: See comments above.

Data: As with export demand we have used our judgement, combined with BEA data, to arrive at sensible estimates.

Intermediate Demand for Imports

Comments: Imports consist of the share of domestic consumption that is not supplied by domestic production.

$$\text{Eq. 17.} \quad m_{t,i} = (1 - d_{t,i}) x_{t,i} \quad \forall i \in I, t \in T$$

Description: See comments above.

Average Prices by Industry

Comments: These aggregated prices are computed for each industry, and are weighted averages of the domestic price and the import price, with the weights consisting of the respective shares in consumption. The price is set to unity in the baseline situation.

$$\text{Eq. 18.} \quad p_{t,i} = d_{t,i} p_{t,i}^d + (1 - d_{t,i}) \bar{p}_{t,i}^w \quad \forall i \in I, t \in T$$

INVESTMENT

We first constructed a measure of the capital stock for each industrial sector for 2006. This stock, less depreciation and plus gross investment gives the capital stock for 2007. Gross investment is determined, sector-by-sector, based on the net of tax rate of return (relative to the return in the base period). For instance, once investment by the agricultural sector has been determined, it is transformed with the help of the capital coefficient matrix into the demand for goods and services for each sector in the economy.⁹

Capital Stock

Comments: The capital stock in time t is the capital stock from the previous period adjusted for depreciation, and augmented by gross investment.

$$\text{Eq. 19.} \quad u_{t,K,i} = u_{t-1,K,i} (1 - \delta_i) + n_{t,i} \quad \forall i \in I, t \in T$$

Description: See comments above.

Data: A complete discussion of the construction of capital stock figures is given in *Texas State Tax Modeling Program: Texas-STAMP* (1999); the same approach and the same data sources are used for Pennsylvania.

⁹ The Capital Coefficient Matrix is a matrix of investments by using industries. It contains distribution ratios of new structures and equipment to using industries from the 1992 BEA capital flow tables.

Gross Investment by Sector of Destination

Comments: The amount of gross investment in any given sector depends on the after-tax rate of return in that sector relative to the return in the base period. The terminology here can be confusing; investment destined for agriculture, for instance, consists of the purchases of goods that will add to the capital stock in the agricultural sector; the goods themselves will mainly come from other sectors (the sectors of source).

$$\text{Eq. 20. } n_{t,i} = \bar{n}_{t,i} \left[\frac{r_{t,K,i} \left(1 - \sum_{g \in GK} \tau_{t,g,K,i}^x \right) u_{t,K,i}}{\bar{r}_{t,K,i} \left(1 - \sum_{g \in GK} \tau_{t,g,K,i} \right) \bar{u}_{t,K,i}} \right]^{\eta^i} \quad \forall i \in I, t \in T$$

Description: Gross investment is the baseline gross investment by industry adjusted to the change in after-tax capital rental rates.

Data: The rate of return is computed as the property-type income for each sector (from BEA) divided by the capital stock (authors' computations). Based on the econometric results from STAMP models estimated for Pennsylvania and elsewhere, we estimated the investment demand elasticity to be about 0.3.

Gross Investment by Sector of Source

Comments: Given that investment has been determined for each sector of destination, this equation allows one to determine who will actually produce the investment goods. This is done with the help of a capital coefficient matrix.

$$\text{Eq. 21. } p_{t,i} \left(1 + \sum_{g \in GS} \tau_{t,g,i}^n \right) cn_{t,i} = \sum_{i' \in I} \beta_{i,i'} n_{t,i'} \quad \forall i \in I, t \in T$$

Description: The gross investment by source in after-tax prices is a function of investment by destination according to the capital coefficient matrix.

Data: Based on the 1992 capital coefficient matrix for the United States from the BEA/Department of Commerce.

GOVERNMENT

Government derives income from a wide range of taxes. It purchases goods and services and makes transfers (such as pensions) to individuals. Some government spending is assumed to remain unchanged even if tax revenues vary; the rest of spending is endogenous, in that it responds to the availability of funds. Notionally, most revenues flow into the Pennsylvania General Fund; they are then used in part to buy goods and services, but some are also transferred to local government units.

Government Income

Comments: This equation adds up government income from multiple sources, including indirect taxes (sales, motor fuels) and direct taxes (income, franchise tax).

Eq. 22.

$$\begin{aligned}
 y_{t,g} = & \sum_{i \in I} \tau_{t,g,i}^v v_{t,i} p_{t,i} + \sum_{i \in I} \tau_{t,g,i}^m m_{t,i} p w_{t,i}^0 + \sum_{h \in H} \sum_{i \in I} \tau_{t,g,i}^c c_{t,i,h} p_{t,i} + \sum_{i \in I} \tau_{t,g,i}^n c n_{t,i,n} p_{t,i} + \sum_{i \in I} \sum_{g' \in G} \tau_{t,g,i}^s c_{t,i,g'} p_{t,i} \\
 & + \sum_{i \in I} \sum_{f \in F} \tau_{t,g,f,i}^x r_{t,f,i} r_{t,f,i}^a u_{t,f,i}^d + \sum_{g' \in G} \sum_{f \in F} \tau_{t,g,f,g'}^x r_{t,f,g'} r_{t,f,g'}^a u_{t,f,g'}^d + \sum_{f \in F} \tau_{t,g,f}^{fh} y_{t,f} + \sum_{h \in H} \tau_{t,h,g}^{pi} a_{t,h}^{hh} + \sum_{h \in H} \tau_{t,h,g}^h a_h^{hh} \\
 & \forall g \in G, t \in T
 \end{aligned}$$

Description: Income by government sector is the sum of taxes on intermediates, imports, consumption, investment, government consumption, factors, income taxes and other household taxes.

Government Endogenous Purchases of Goods and Services

Comments: Spending on these items is assumed to take a fixed fraction of total government receipts (from taxes and net intergovernmental transfers, less government savings). The endogenous sectors are state spending on education, health, safety, transport and “other,” and local spending on education and health.

Eq. 23.

$$p_{t,i} \left(1 + \sum_{g \in GS} \tau_{t,g,i}^g \right) c g_{t,i,g} = \alpha_{i,g} \left(y_{t,g} + \sum_{g' \in G} b_{t,g,g'} - \sum_{g' \in G} b_{t,g',g} + b_{t,ussstx,g} - \sum_{h \in H} w_{t,h,g} a_{t,h}^n \tau_{t,h,g}^{pc} - \bar{s}_{t,g} \right) \quad \forall i \in I, g \in GN, t \in T$$

Description: The government spending in after-tax prices computed according to their share of government income plus net inter-government transfers less government savings and transfer payments. Note that only state and local governments are endogenous in the model.

Data: The shares of spending going to these sectors are based on a careful analysis of Pennsylvania government budget and financial reports.

Government Endogenous Rental of Factors

Comments: As in the case of goods and services, government is also assumed to devote a fixed share of its total spending to the purchase of labor and capital services for those sectors considered to be endogenous.

Eq. 24.

$$u_{t,f,g}^d r_{t,f}^a r_{t,f,g} = \alpha_{f,g} \left(y_{t,g} + \sum_{g' \in G} b_{t,g,g'} - \sum_{g' \in G} b_{t,g',g} + b_{t,ussstx,g} - \sum_{h \in H} w_{t,h,g} a_{t,h}^n \tau_{t,h,g}^{pc} - \bar{s}_{t,g} \right) \quad \forall f \in F, g \in GN, t \in T$$

Description: The government factor demand is computed according to the share of each government in total government spending, including net inter-government transfers, less savings and transfer payments.

Government Infrastructure Capital Stock

Comments: The government adds to its infrastructure capital stock through its spending on the government transportation sector, STTRAN.

$$\text{Eq. 25.} \quad gk_{t+1} = gk_t(1 - \delta) + \sum_{g \in G} b_{t+1, STTRAN, g} - \sum_{g \in G} b_{t+1, g, STTRAN} + \sum_{g \in G} b_{t+1, LOTRAN, g} \quad \forall t \in T$$

Description: The infrastructure capital stock for the current year is the infrastructure for the previous year, less depreciation plus the net spending on transportation by state and local governments.

Data: The data for government infrastructure capital stock is based on national data from the BEA.

Government Savings

Comments: Government saving is a residual, consisting of revenue less spending.

$$\text{Eq. 26.} \quad s_{t, g} = y_{t, g} - \sum_{i \in I} c g_{t, i, g} p_{t, i} \left(1 + \sum_{g \in GS} \tau_{t, g, i}^g \right) - \sum_{f \in F} u_{t, f, g}^d r_{t, f, g} r_{t, f}^a \left(1 + \sum_{g' \in GF} \tau_{t, f, g', g}^x \right) - \left(\sum_{h \in H} w_{t, h, g} a_{t, h}^n \tau_{hg}^{pc} \right) - \sum_{g \in G} b_{t, g', g} + b_{t, USSSTX, g} + \sum_{g' \in G} b_{t, g, g'} \quad \forall g \in G, t \in T$$

Description: Government savings is the residual from government income, after spending and factor rental, transfer payments, plus net inter-governmental transfers.

Distribution of Taxes to Spending and Transfers

Comments: Tax units, in this case those sectors collecting revenues, distribute some of their receipts to spending units, and others directly in the form of transfers to households. The matrix IGTD (in the miscellaneous input file) identifies which units pass on their revenues to other spending units, and the flows are recorded in this equation.

$$\text{Eq. 27.} \quad b_{t, g', g} = \mu_{t, g', g} \left(y_{t, g} - \left(\sum_{h \in H} w_{t, h, g} a_{t, h}^n \tau_{t, h, g}^{pc} - \bar{s}_{t, g} \right) \right) \quad \forall g, g' \in G$$

Description: The intra-fund accounting to distribute the government income, less transfer payments and savings.

Data: This equation is based on institutional arrangements in place in PA.

Endogenous Distribution of PA Funds

Comments: This equation details the flows from state funds to state spending sectors and from state spending sectors to local spending sectors.

$$\text{Eq. 28.} \quad b_{t,g,g'} = \mu_{t,g,g'} \left(\sum_{g''} b_{t,g',g''} + w_{g',INVEST} + w_{g',ROWSCT} \right) \quad \forall g, g' \in G$$

Description: Some funds are fixed to the original share.

Data: Based on an analysis of the current pattern of spending in PA.

State Personal Income

Comments: This equation defines state personal income as earnings (from labor and capital) plus transfer payments.

$$\text{Eq.29.} \quad y_t^s = \sum_{h \in H} y_{t,h} + \sum_{h \in H} \sum_{g \in G} w_{t,h,g} a_{t,h}^n \tau_{h,g}^{pc} \quad \forall t \in T$$

Description: State personal income is the sum of household income and government transfer payments.

MODEL CLOSURE

Labor Market Clearing

Comments: Labor supply equals labor demand. For this to occur, the wage rate must adjust to bring about this market clearing.

$$\text{Eq. 30.} \quad \sum_{h \in H} a_{t,h}^w = \left(\sum_{z \in Z} u_{t,L,z}^d \right) \varepsilon_t \quad \forall t \in T$$

Description: Total working households equals the sum of private employment and government employment.

Capital Market Clearing

Comments: Capital markets also clear for each sector. In other words, demand for capital by industries equals supply of capital.

$$\text{Eq. 31.} \quad u_{t,K,i}^s = u_{t,K,i}^d \quad \forall i \in I, t \in T$$

Description: See comments above.

Goods Market Clearing

Comments: Domestic demand (intermediate, consumer, government and investment demand) plus exports less imports must equal domestic supply.

$$\text{Eq. 32.} \quad q_{t,i} = x_{t,i} + e_{t,i} - m_{t,i} \quad \forall i \in I, t \in T$$

Description: See comments above.

Domestic Demand Defined

Comments: These equations define domestic demand for each sector.

$$\text{Eq.33.} \quad x_{t,i} = v_{t,i} + \sum_{h \in H} c_{t,i,h} + \sum_{g \in G} c g_{t,i,g} + c n_{t,i} \quad \forall i \in I, t \in T$$

Description: Domestic demand is the sum of intermediate demand, household consumption, government consumption and investments.

PIT for Non Income Tax Units

Comments: This equation sets the personal income tax for non-income tax units to zero; this is a technicality that ensures the solution to the model does not create income tax revenue in an inappropriate place.

$$\text{Eq.34.} \quad t_{t,g,h} = 0 \quad \forall h \in H, g \notin GI, t \in T$$

Set Intergovernmental Transfers to Zero if Not in Original SAM

Comments: This is another housekeeping equation that ensures the solution to the model does not create inter-governmental transfers where they should not occur.

$$\text{Eq.35.} \quad b_{t,g,g'} = 0 \quad \forall g, g' \in G, t \in T \quad \text{where } \bar{b}_{gg'} = 0$$

Federal Social Security Transfers to PA

Comments: Transfers paid to PA households from the Federal social security system are assumed to be mainly determined by the number of households in the state.

$$\text{Eq.36.} \quad b_{t,h,USSTX} = \bar{b}_{t,h,USSTX} \times \left(\frac{\overline{a_{t,h}^n}}{a_{t,h}} \right)$$

Description: Transfer payments are adjusted by the change in nonworking households.

Fix Exogenous Federal Transfers to Households

Comments: Federal transfers to households are assumed to vary with the number of households in the state.

$$\text{Eq.37.} \quad b_{t,h,\text{USNOND}} = \bar{b}_{t,h,\text{USNOND}} \times \left(\frac{a_{t,h}^n}{a_{t,h}^n} \right)$$

Description: Transfer payments are adjusted by the change in nonworking households.

Fix Goods and Services Demand by Exogenous Government Units

Comments: The purchases of goods and services by some government sectors are considered to be exogenous to the model. This equation fixes these values.

$$\text{Eq. 38.} \quad c g_{t,i,g} = \bar{c} g_{t,i,g} \quad \forall i \in I, g \in GX, t \in T$$

Fix Factor Rentals Paid by Exogenous Government Units

Comments: The purchases of the services of labor and capital are considered to be exogenous to the model. This equation fixes these values.

$$\text{Eq. 39.} \quad u_{t,f,g}^d = \bar{u}_{t,f,g}^d \quad \forall f \in F, g \in GX, t \in T$$

Fix Intersectoral Wage Differentials

Comments: Although wage rates differ from sector to sector, these differentials are assumed to remain fixed, as set by this equation. Household labor supply responds to overall wage rates, and not to the wage rates in any particular sector.

$$\text{Eq. 40.} \quad r_{t,L,i} = \bar{r}_{t,L,i} \quad \forall i \in I, t \in T$$

Fix Government Rental Rate for Capital to Initial Level

Comments: For PA STAMP, we have set these rental rates to zero, in the absence of viable information about the rental rates paid by government on the capital that it uses. However, the relevant equations are included, and so government rental rates could be incorporated in a future version of the model.

$$\text{Eq. 41.} \quad r_{t,K,g} = \bar{r}_{t,K,g} \quad \forall g \in G, t \in T$$

Fix Economy Wide Scalar for Capital

Comments: The model allows both for an overall cost of capital, and sector-specific returns. This equation sets the overall scalar to its original level, so that only the sector-specific returns vary endogenously.

$$\text{Eq. 42.} \quad r_{t,K}^a = \bar{r}_{t,K}^a \quad \forall f \in F, t \in T$$

Set Transfer Payments to Zero if Originally So

Comments: This equation ensures that if transfer payments to households were zero in the original social accounting matrix, they remain at zero.

$$\text{Eq. 43.} \quad w_{t,h,g} = 0 \quad \forall h \in H, g \in GWX, t \in T \quad \text{where} \quad \bar{w}_{t,h,g} = 0$$

OBJECTIVE FUNCTION

Comments: This equation measures utility over the entire period of the dynamic model as measured by the sum of state personal income discounted. The variable is of interest in its own right. However it also provides a convenient variable for GAMS to maximize (or minimize), because it is an unrestricted variable without a subscript.

$$\text{Eq. 44.} \quad U = \sum_{t \in T} \beta_t \text{state}y_t \quad t \in T$$

Description: Utility is defined as the net present value of future state personal income levels.

ELASTICITY ASSUMPTIONS FOR PA STAMP

For the model to work, one has to introduce values for the relevant elasticities. These are drawn from the existing literature, as follows:

ETAM: Import elasticity with respect to domestic price for producers' purchase of intermediates. Most of the data on elasticities are taken from Reinert, Roland-Holst, and Shiells. The two most recent are Reinert and Roland-Holst (1992)¹⁰ and Roland-Holst, Reinert and Shiells (1994)¹¹.

In the first study, the authors estimate an Armington model for 163 mining and manufacturing sectors. Two-thirds of the elasticities were positive and statistically significant, ranging from a low of 0.13 for chocolate to 3.49 for wine, brandy and brandy spirits. The second study looked at the impact of NAFTA. In this study many of the aggregate industries had an elasticity of 1.50. Since import data for goods between states is almost impossible to obtain, we made some assumptions and used 1.50 for most industries and a slightly lower elasticity of 0.50 for a handful of less traded industries such as service industries.

While these elasticities are slightly higher than the literature on national trade, we believe that goods in a state are more price-sensitive to goods in the Rest of the World (including other states) than national goods. Therefore, we converted the elasticities to a domestic share elasticity for each industry using the following equation. $ETAD = ETAM * IMPORT / (DOM. DEMAND * DOM. SUPPLY SHARE OF DOM. DEMAND)$. The estimates for this elasticity were taken from the literature.

ETAE: Export elasticity with respect to domestic price for the sale producers' goods. Used in the export demand equation. The NAFTA study was also helpful with exports. We used an elasticity of 1.65 for industries which had an import elasticity of 1.50 and an export elasticity of 0.65 for those which had an import elasticity of 0.50.

SIGMA: Elasticity of substitution between capital and labor. Values in the literature range between 0.15 and 1.809 for industries with the majority close to 1, and we have used values of

¹⁰K.A. Reinert and D.W. Roland-Holst. "Armington Elasticities for United States Manufacturing Sectors". *Journal of Policy Modeling*. 14, no.5 (1992): 631-639.

¹¹ D.W. Roland-Holst K.A. Reinert, and C.R. Shiells. "A General Equilibrium Analysis of North American Economic Integration". *Modeling Trade Policy: Applied General Equilibrium Assessments of North American Free Trade*. Cambridge Univ. Press (1994): 47-82.

0.90 for industries with substantial substitution and 0.8 in other cases (as shown in Table 2). This measurement is used to calculate RHO, which is the exponent in the production function. The equation is: $RHO = (1 - SIGMA)/SIGMA$.

Table 2. Industry Elasticities					
	ETAM	ETAE	ETAY	ETAOP	SIGMA
AGRICF	1.50	-1.65	1.00	-1.00	0.90
MINING	1.50	-1.65	1.00	-1.00	0.80
CONSTR	1.50	-1.65	1.00	-1.00	0.90
FOODPR	1.50	-1.65	1.00	-1.00	0.90
APPARL	1.50	-1.65	1.00	-1.00	0.90
MFRCON	1.50	-1.65	1.00	-1.00	0.80
PPAPER	1.50	-1.65	1.00	-1.00	0.80
CHEMIC	1.50	-1.65	1.00	-1.00	0.80
ELECTR	1.50	-1.65	1.00	-1.00	0.90
MVOTRA	1.50	-1.65	1.00	-1.00	0.90
METALS	1.50	-1.65	1.00	-1.00	0.80
MACHIN	1.50	-1.65	1.00	-1.00	0.80
INSTRU	1.50	-1.65	1.00	-1.00	0.90
MFROTH	1.50	-1.65	1.00	-1.00	0.90
TRANSP	1.50	-1.65	1.00	-1.00	0.90
COMMUN	1.50	-1.65	1.00	-1.00	0.90
UTILIT	1.50	-1.65	1.00	-1.00	0.80
WHOLSA	0.50	-0.65	1.00	-1.00	0.90
RETAIL	0.50	-0.65	1.00	-1.00	0.90
BANKNG	1.50	-1.65	1.00	-1.00	0.90
INSURS	1.50	-1.65	1.00	-1.00	0.90
REALST	1.50	-1.65	1.00	-1.00	0.90
REPSVC	1.50	-1.65	1.00	-1.00	0.80
BSVCES	1.50	-1.65	1.00	-1.00	0.80
ENTRHO	0.50	-0.65	1.00	-1.00	0.80
HEALTH	0.50	-0.65	1.00	-1.00	0.80
OTHSVC	0.50	-0.65	1.00	-1.00	0.80
USNOND	0	0	0	0	0
USDEFF	0	0	0	0	0
STGGSP	0	0	0	0	0
STEDUC	0	0	0	0	0
STHELT	0	0	0	0	0
STPBSF	0	0	0	0	0
STTRAN	0	0	0	0	0
STOTHS	0	0	0	0	0
LOEDUC	0	0	0	0	0
LOHELT	0	0	0	0	0

LOPBSF	0	0	0	0	0
LOTRAN	0	0	0	0	0
LOOTHS	0	0	0	0	0

The following elasticities are used in household-specific equations:

ETAPIT: Labor supply elasticity with respect to income taxes. This elasticity appears as an exponent in the labor supply equation. Measurements were based on estimates taken from the literature. The labor supply elasticities (ETARA) are widely divergent in the literature and suffer from a lack of disaggregation. They range from close to zero to 2.3 for net wages, with rather high positive values for women, particularly married woman. This means that the *tax* elasticities are negative. There is some evidence of greater (absolute) tax elasticities at higher income levels, which is why we assume a graduated scale from -0.15 for the lowest income category to -0.35 in the top category (see Table 3).¹²

ETATP: Household response to transfer payments. The transfer payment elasticities reflect a study by Robins (1985) on the effects of a negative income tax (NIT). It is also a reflection of the observation that income received by upper income groups is on average largely unaffected by transfer payments.

ETAYD: Responsiveness of immigration to after tax income. Not much literature exists that ties migration to disposable income or unemployment. Studies by Bartik (1991), Valiant (1994), and Treyz et al. (1993) put the range of responses to a change in wage rates at between 0.835 and 2.39. We used these as a basis for our after tax earnings elasticities. This elasticity appears in the population equation.

ETAU: Responsiveness of immigration to unemployment. We made some assumptions based on the responsiveness to employment elasticities in the literature.

ETAMH: Income elasticity of demand for imports by household. This elasticity appears in the household import equation.

¹² Note that $ETAPIT = -ETARA (t/(1-t))$, where t is the income tax rate.

Table 3. Household-Related Elasticities						
	ETAPIT	ETATP	ETARA	ETAYD	ETAU	ETAMH
LESS10	-0.15	-0.05	0.17	1.30	-0.80	0.70
LESS25	-0.18	-0.05	0.17	1.50	-0.80	0.70
LESS50	-0.20	-0.04	0.20	1.60	-0.80	0.70
LESS75	-0.25	-0.04	0.30	1.80	-0.80	0.70
LES100	-0.25	-0.03	0.40	2.00	-0.80	0.70
LES150	-0.30	-0.03	0.50	2.10	-0.80	0.70
MOR150	-0.35	-0.02	0.50	2.30	-0.80	0.70

APPENDIX: DEFINITIONS AND GLOSSARY OF TERMS

Summary of Set Names

Sets	Dimension	Math	GAMS
Factors	2	$f \in F$	F
Governments - All	39	$g \in G$	G
Governments - Factor Taxes	6	$g \in GF$	GF
Governments - Per Household Taxes	8	$g \in GH$	GH
Governments - Income Taxes	2	$g \in GI$	GI
Governments - Capital Income Taxes	6	$g \in GK$	GK
Governments - Endogenous Spending	16	$g \in GN$	GN
Governments - Sales or Excise Taxes	11	$g \in GS$	GS
Governments - Endogenous Transfer Payments	1	$g \in GWN$	GWN
Governments - Exogenous Transfer Payments	4	$g \in GWX$	GWX
Governments - Exogenous Spending	6	$g \in GX$	GX
Households	7	$h \in H$	H
Industries	27	$i \in I$ or $j \in I$	I
All Social Accounting Matrix Accounts	77	$z \in Z$	Z

Summary of Parameter Names

Parameters	Dimension	Math	GAMS
Input Output Coefficients	77 x 77	-	A(Z,Z1)
Domestic Input Output Coefficients	27 x 27	α_{ii}	AD(Z,Z1)
Government Spending Shares of Net Income	39 x 39	α_{ie}, α_{fe}	AG(Z,G)
Factor Share Exponents in Production Function	2 x 27	α_{fi}	ALPHA(F,I)
Initial Shares of Consumption	27 x 7	α_{ih}	ALPHA(I,H)
Deductibility of Taxes	3 x 3	α_{ee}^t	ATAX(G,G1)
Income Elasticities of Demand	27 x 7	β_{ih}	BETA(I,H)
Capital Coefficient Matrix	27 x 27	β_{ii}	CCM(I,J)
Depreciation Rate	27	δ_i	DEPR(I)
Export Price Elasticities	27	η_i^e	ETA(E,I)
Domestic Demand Elasticity	27	η_i^d	ETAD(I)
Investment Supply Elasticity	1	η_i	ETAI
L Supply Elasticity with respect to Average Wage	7	η_h^{ls}	ETARA(H)
Labor Supply Elasticity with respect to TP's ¹³	7	η_h^{tp}	ETATP(H)
Labor Supply Elasticity with respect to Taxes	7	η_h^{PIT}	ETAPIT(H)
Responsiveness of In-Migration to Unemployment	7	η_h^u	ETAU(H)
Responsiveness of In-Migration to Disp. Income	7	η_h^{yd}	ETAYD(H)
Production Function Scale	27	γ_i	GAMMA(I)
Types of Inter-Government Transfers	39 x 39	-	IGTD(G,G1)
Correction Factor between Households and Jobs	1	ε	JOBCOR
Price Elasticities of Demand	27 x 27	λ_{ii}	LAMBDA(I,J)
Miscellaneous Industry Parameters	27 x 10	-	MISC(Z,*)
Income Tax Table Data in Input File	7 x 8	-	MISCG(G,H,*)
Miscellaneous Household Parameters	7 x 8	-	MISCH(H,*)
Natural Rate of Population Growth	7	π_h	NRPG(H)
Substitution Exponent in Production Function	27	ρ_i	RHO(I)
Social Accounting Matrix	77 x 77	$\sigma_{...}$	SAM(Z,Z1)
Consumption Sales and Excise Tax Rates	9 x 27	τ_{oi}^c	TAUC(G,I)
Factor Tax Rates	5 x 2 x 77	τ_{ofz}	TAUF(G,F,Z)
Factor Taxes applied to Factors	5 x 2	-	TAUFF(GF,G)
Employee Portion of Factor Taxes	5 x 2	τ_{of}	TAUFH(G,F)
Experimental Factor Tax Rates	5 x 2 x 77	τ_{ofz}^x	TAUFX(G,F,Z)
Government Sales and Excise Tax Rates	9 x 27	τ_{ei}^g	TAUG(G,I)
Household Taxes other than PIT	1 x 7	τ_{oh}	TAUH(G,H)
Investment Sales and Excise Tax Rates	9 x 27	τ_{oi}^n	TAUN(G,I)
Sales and Excise Tax Rates	9 x 27	τ_{oi}^q	TAUQ(G,I)
Intermediate Good Sales and Excise Tax Rates	9 x 27	τ_{oi}^v	TAUV(G,I)
Tax Bracket Base Amount	2 x 7	τ_{oh}^b	TAXBASE(G,H)
Tax Bracket Minimum Taxable Earnings	2 x 7	τ_{eh}^d	TAXB(M,G,H)
Tax Constant to Correct Calculated to Observed	2 x 7	τ_{oh}^c	TAXCVC(G,H)
Tax Deduction other than Standard and other PIT	2 x 7	τ_{oh}^o	TAXOD(G,H)
Percentage Itemizing	2 x 7	τ_{eh}^i	TAXPI(G,H)
Tax Destination Shares	39 x 39	$\mu_{\sigma\sigma'}$	TAXS(G,G1)
Tax Deduction for Standard Deductions	2 x 7	τ_{oh}^s	TAXSD(G,H)
Percent of Households Receiving TP's	7 x 6	τ_{hp}^{pc}	TPC(H,G)

¹³ TP is abbreviation for transfer payments.

Summary of Variable Names

Variables	Dimension	Math	GAMS
Public Consumption	27 x 39	c_{ig}	CG(I,G)
Private Consumption	27 x 7	c_{ih}	CH(I,H)
Gross Investment by Sector of Source	27	c_{in}	CN(I)
Consumer Price Index	7	p_h	CPI(H)
Exports	27	e_i	CX(I)
Domestic Share of Domestic Consumption	27	d_i	D(I)
Domestic Demand	27	x_i	DD(I)
Domestic Supply	27	q_i	DS(I)
Sectoral Factor Demand	2 x 77	u_{fi}^d, u_{fg}^d	FD(F,Z)
Number of Households	7	a_h	HH(H)
Number of Non-Working Households	7	a_h^n	HN(H)
Number of Working Households	7	a_h^w	HW(H)
Household Out-Migration	7	a_h^o	MO(H)
Household In-Migration	7	a_h^i	MI(H)
Inter-Governmental Transfers	37 x 37	$B_{gg'}$	IGT(G,G1)
Capital Stock	27	u_{Ki}^s	KS(I)
Imports	27	m_i	M(I)
Gross Investment by Sector of Destination	27	n_i	N(I)
Net Capital Inflow	1	z	NKI
Aggregate Price	27	p_i	P(I)
Aggregate Price including Sales/Excise Taxes	27	p_i^c	PC(I)
Domestic Producer Price	27	p_i^d	PD(I)
Per Household Personal Income Taxes	2 x 7	t_{gh}	PIT(G,H)
Producer Price Index	1	p	PPI
Value Added Price	27	p_i^{va}	PVA(I)
World Price (Rest of US and Rest of World)	27	p_i^w	PW(I)
Sectoral Factor Rental Rates	2 x 27	r_{fi}, r_{fg}	R(F,I)
Economy Wide Scalar for Factor Rental Rates	2	r_f^a	RA(F)
Government Savings	39	s_g	S(G)
Private Savings	7	s_h	S(H)
State Personal Income	1	q	SPI
Transfer Payments	7 x 39	w_{hg}	TP(H,G)
Intermediate Goods	27	v_i	V(I)
Factor Income	2	y_f	Y(F)
Government Income	39	y_g	Y(G)
Household Income	7	y_h	Y(H)
Household after Tax Income including TP's	7	Y_h^d	YD(H)

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